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BIOGRAPHY.

BOLYAI FARKAS. [WOLFGANG BOLYAI.]

BY DR. GEORGE BRUCE HALSTED.

FOR the treatment of parallels, what Frischauf calls "das anschaulichste Axiom," is due to the researches of Bolyai Farkas. He gives it in his "Kurzer Grundriss eines Versuchs" etc., p. 46, as follows: "Koennten jede 3 Punkte, die nicht in einer Geraden sind, in eine Sphaere fallen; so waere das Eucl. Ax. XI. bewiesen." Thus the space whose every three points are co-straight or concyclic is Euclidean.

But in his Autobiography written in Magyar, of which my forthcoming life of the Bolyais contains the first translation ever made, he says: "Yet I was not satisfied with my attempts to prove the Problem of Parallels, which was ascribable to the long discontinuance of my studies, or more probably it was due to myself that I drove this problem to the point which robbed my rest, deprived me of tranquility."

Hitherto what was known of the Bolyais came wholly from the published works of the father, Bolyai Farkas, and from a brief article by Architect Fr. Schmidt of Budapest, "Aus dem Leben zweier ungarischer Mathematiker, Johann und Wolfgang Bolyai von Bolya. Grunerts Archiv, Bd. 48, 1868, p. 217.

In two communications sent me in September and October, 1895, Herr Schmidt has very kindly and graciously put at my disposal the results of his subsequent researches which I will here reproduce. But meantime I have from entirely another source come most unexpectedly into possession of original documents so extensive, so precious that I have determined to issue them in a

BOLYAI FARKAS. [WOLFGANG BOLYAI.]

separate volume devoted wholly to the life of the Bolyais ; but these are not used in the sketch here given.

Bolyai Farkas was born February 9th, 1775, at Bolya in that part of Transylvania (Erdély) called Székelyföld. He studied first at Enyed, afterward at Klausenburg (Kolozsvár), then went with Baron Simon Kemény to Jena and afterward to Goettingen. Here he met Gauss, then in his 19th year, and the two formed a friendship which lasted for life.

The letters of Gauss to his friend were sent by Bolyai in 1855 to Professor Sartorius von Walterhausen, then working on his biography of Gauss. Everyone who met Bolyai felt that he was a profound thinker and a beautiful character.

Benzenberg said in a letter written in 1801 that Bolyai was one of the most extraordinary men he had ever known.

He returned home in 1799, and in January, 1804, was made professor of mathematics in the Reformed College of Maros-Vásárhely. Here for 47 years of active teaching he had for scholars nearly all the professors and nobility of the next generation in Erdély.

Sylvester has said that mathematics is poesy.

Bolyai's first published works were dramas.

His first published book on mathematics was an arithmetic : Az arithmetica eleje. 8vo. I—XVI, 1—162 pp. The copy in the library of the Reformed College is enriched with notes by Bolyai János.

Next followed his chief work, to which he constantly refers in his later writings. It is in Latin, two volumes, 8vo. with title as follows: TENTAMEN | JUVENTUTEM STUDIOSAM | IN ELEMENTA MATHESEOS PURÆ, ELEMENTARIS AC | SUBLIMIORIS, METHEDO INTUITIVA, EVIDENTIA— | QUE HUIC PROPRIA, INTRODUCENDI. | CUM APPENDICE TRIPLICE. |

Auctore Professore Matheseos et Physices Chemiæque | Publ. Ordinario. | Tomus Primus. | *Maros Vásárhelyini.* 1832. | Typis Collegii Reformatorum per JOSEPHUM, et | SIMEONEM KALI de felsö Vist. | At the back of the title : Imprimatur. | M. Vásárhelyini Die | 12 Octobris 1829. |

The now world renowned Appendix by Bolyai János was an afterthought of the father, who prompted the son not 'to occupy himself with the theory of parallels,' as Staeckel says, but to translate from the German into Latin a condensation of his treatise, of which the principles were discovered and properly appreciated in 1823, and which was given in writing to J. W. von Eckwehr in 1825.

The father, without waiting for Vol. II., inserted this Latin translation, with separate paging (1—26), as an Appendix to his Vol. I., where, counting a page for the title and a page 'Explicatio signorum,' it has twenty-six numbered pages, followed by two unnumbered pages of Errata.

The treatise itself, therefore, contains only twenty-four pages—the most extraordinary two dozen pages in the whole history of thought !

Milton received but a paltry 5 pounds for his Paradise Lost ; but it was at least plus 5. Bolyai Janos, as we learn from Vol. II., p. 384 of '*Tentamen*,' contributed for the printing of his eternal 26 pages, 104 florins 54 kreuzers.

That this Appendix was finished considerably before the Vol. I., which it follows, is seen from the references in the text, breathing a just admiration for the Appendix and the genius of its author,

Thus Bolyai Farkaš says, p. 452 : Elegans est conceptus *similium*, quem J. B. *Appendicis Auctor* dedit; again, p. 489: *Appendicis Auctor*, rem acumine singulari aggressus, Geometriam pro omni casu absolute veram posuit; quamvis e magna molé, tantum summe necessaria, in Appendice hujus tomi exhibuerit, multis (ut tetraedri resolutione generali, pluribusque aliis disquisitionibus elegantibus) brevitatis studio omissis. And the volume ends as follows, p. 502: Nec operae pretium est plura referre ; quum res tota ex altiori contemplationis puncto, in ima prenetranti oculo, tractetur in Appendice sequente, a quovis fideli veritatis purae alumno digna legi.

The father gives a brief resumé of the results of his own determined, lifelong, desperate efforts to do that at which Saccheri, J. H. Lambert, Gauss also had failed, to establish Euclid's theory of parallels *a priori*.

He says, p. 490: "tentamina idcirco quae olim feceram, breviter exponenda veniunt ; ne saltem alius quis operam eandem perdat." He anticipates J. Delboeuf's "Prolégomènes philosophiques de la géométrie et solution des postulato," with the full consciousness in addition that it is *not* the solution,—that the final solution has crowned not his own intense efforts, but the genius of his son.

This son's Appendix which makes all preceding space only a special case, only a species under a genus, and so requiring a descriprive adjective, *Euclidean*, this wonderful production of pure genius, this strange Hungarian flower was saved for the world after more than thirty-five years of oblivion, by the rare erudition of Professor Richard Baltzer of Dresden, afterward professor in the University of Giessen. He it was who first did justice publicly to the works of Lobachevski and Bolyai.

Incited by Baltzer, 1866, J. Hoüel issued a French translation of Lobachevski's Theory of Parallels and in a note to his Preface says: "M. Richard Baltzer, dans la seconde édition de ses excellents *Éléments de Géométrie*, a, le premier, introduit ces notions exactes à la place qu'elles doivent occuper." Honor to Baltzer ! But alas ! father and son were already in their graves !

Fr. Schmidt in the article cited (1868) says: "It was nearly forty years before these profound views were rescued from oblivion, and Dr. R. Baltzer, of Dresden, has acquired imperishable titles to the gratitude of all friends of science as the first to draw attention to the works of Bolyai, in the second edition of his excellent Elemente der Mathematik (1866-67). Following the steps of Baltzer, Professor Hoüel, of Bordeaux, in a brochure entitled: Essai critique sur les principes fondamentaux de la Géométrie élémentaire, has give extracts from Bolyai's book, which will help in securing for these new ideas the justice they merit."

The father refers to the son's Appendix again in a subsequent book, Ürtan elemei Kezdöknek [Elements of the science of space for beginners] (1850-51), pp. 48. In the College are preserved three sets of figures for this book, two by the

author, and one by his grandson, a son of János. The last work of Bolyai Farkas, the only one composed in German, is entitled: Kurzer Grundriss eines Versuchs

I. Die Arithmetik, durch zvekmässig Konstruirte Begriffe, von eingebildeten und unendlich-kleinen Grössen gereinigt, anschaulich und logisch-streng darzustellen.

II. In der Geometrie, die Begriffe der geraden Linie, der Ebene, des Winkels allgemein, der winkellosen Formen, und der Krummen, der verschiedenen Arten der Gleichheit u.d.gl. nicht nur scharf zu bestimmen; sondern auch ihr Seyn im Raume zu beweisen: und da die Frage, *ob zwey von der dritten geschnittene Geraden, wenn die summe der inneren Winkel nicht* $=2R$, *sich schneiden oder nicht?* Niemand auf der Erde ohne ein Axiom (wie Euklid das XI) aufzustellen, beantworten wird; die davon unabhängige Geometrie abzusondern; und eine auf die *Ja*-Antwort, andere auf das *Nein* so zu bauen, das die Formeln der letzten, auf ein Wink auch in der ersten gültig seyen.

Nach ein lateinischen Werke von 1829, M. Vásárhely, und eben daselbst gedruckten ungrischen.

Maros Vásárhely 1851. 8vo. pp. 88.

In this book he says, referring to his son's Appendix: "Some copies of the work published here were sent at that time to Vienna, to Berlin, to Goettingen. . . . . From Goettingen the giant of mathematics, who from his pinnacle embraces in the same view the stars and the abysses, wrote that he was surprised to see accomplished, what he had begun, only to leave it behind in his papers." This refers to 1832. The only other record that Gauss ever mentioned the book is a letter from Gerling written October 31st, 1851, to Wolfgang Bolyai on receipt of a copy of 'Kurzer Grundriss.' Gerling, a scholar of Gauss, had been from 1817 Professor of Astronomy at Marburg. He writes: "I do not mention my earlier occupation with the theory of parallels, for already in the year 1810—1812 with Gauss, as earlier as 1809 with J. F. Pfaff I had learned to perceive, how all previous attempts to prove the Euclidean axiom had miscarried. I had then also obtained preliminary knowledge of your works, and so, when I first [1820] had to print something of my view thereon, wrote it exactly so, as it yet stands to read on page 187 of the latest edition.

We had about this time [1819] here a law professor Schweikart, who was formerly in Charkow, and had attained to similar ideas, since without help of the Euclidean axiom he developed in its beginnings a geometry which he called Astralgeometry. What he communicated to me thereon, I sent to Gauss, who then informed me, how much farther already had been attained on this way and later also expressed himself about the great acquisition, which is offered to the few expert judges in the Appendix to your book."

The 'latest edition' mentioned appeared in 1851, and the passage refarred to is: "This proof [of the parallel-axiom] has been sought in manifold ways by acute mathematicians, but yet until now not found with complete sufficiency. So long as it fails, the theorem, as all founded on it, remains a hypothesis, whose

validity for our life indeed is sufficiently proven by *experience*, whose *general, necessary exactness* however could be doubted without absurdity.''

Alas! that this feeble utterance should have seemed sufficient for more than thirty years to the associate of Gauss and Schweikart, the latter certainly one of the independent discoverers of the non-Euclidean geometry. But then since neither of these sufficiently realized the transcendent importance of the matter to publish any of their thoughts on the subject, a more adequate conception of the issues at stake could scarcely be expected of the scholar and colleague. How different with Bolyai János and Lobachévski, who claimed at once, unflinchingly, that their discovery marked an epoch in human thought so momentous as to be unsurpassed by anything recorded in the history of philosophy or of science, demonstrating as had never been proven before the supremacy of pure reason at the very moment of overthrowing what had forever seemed its surest possession, the axioms of geometry.

Austin, Texas, December 16th, 1895.

THE DUPLICATION OF THE NOTATION FOR IRRATIONALS.

By JOS. V. COLLINS, Ph. D., State Normal School, Stevens Point, Wisconsin.

Authorities agree in crediting Rudolff (1525), the German cossist, with the introduction of the radical sign, $\surd$, not precisely as we use it, but one such mark for a square root, three for a cube, and two for a fourth root. Cantor thinks it probably originated from a West-Arabian custom of using dots, by makings *lines* of the dots, and connecting them in the making by lighter lines. These dots in turn originated, it is thought, in the use of the letter, dschim, the first in the Arabian word for *root*. Rudolff was followed by Stifel in the employment of this notation, and afterwards Girard (1633) changed it to the present form. By the middle of the 17th century the mark had come into general use. The exponent notation, though first used by Rudolff and Stifel in a crude form, was employed as we now have it for integral values of the exponents by Descartes. Soon after, Wallis, in his *arithmetica infinitorum* (1656), interpreted and used the simpler forms of fractional exponents, though Stevin (1585) had suggested the meaning to be assigned them. Then in 1676 Newton wrote to Oldenburg "since algebraists write a^2, a^3, a^4, etc., for aa, aaa, $aaaa$, etc., so I write $a^{\frac{1}{2}}$, $a^{\frac{3}{2}}$, $a^{\frac{5}{3}}$, for $\surd a$, $\surd a^3$, $\surd e.a^5$." Newton went further in connection with his binomial theorem, and generalized this use of exponents into the exponential function. The question naturally arises why was it that the old notation for roots was not replaced by the new as had been done in numerous instances before? Doubtless the best